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2978. Proposed by C. N. SCHMALL, New York City.

Given the equation of the general cubic $f(x, y) \equiv ax^3 + 3bx^2y + 3cxy^2 + dy^3 + a_1x^2 + 2b_1xy + c_1y^2 + a_2x + b_2y + c_2 = 0$, show that the three asymptotes of the curve will be concurrent if

$$\begin{vmatrix} a & b & a_1 \\ b & c & b_1 \\ c & d & c_1 \end{vmatrix} = 0.$$

2979. Proposed by V. M. SPUNAR, Chicago, Ill.

If a tangent be drawn from a variable point of an ellipse of length equal to n times the semi-conjugate diameter at the point, the locus of its extremity will be a concentric ellipse with semi-axes equal to $a\sqrt{n^2+1}$, $b\sqrt{n^2+1}$.

2980. Proposed by J. ROSENBAUM, Milford, Conn.

Locate a point such that the sum of its distances from the vertices of a given polygon shall be a minimum.

SOLUTIONS.

2876 [1921, 89]. Proposed by H. S. UHLER, Yale University.

Let a, d, n, and r denote respectively the refracting angle of a prism, the total deviation of the ray (in a principal plane) produced by the prism, the relative index of refraction of the material of the prism with respect to the single surrounding medium, and the angle of refraction at the first or incidence face of the prism.

(*a*) Deduce the (new) formula given below. (*b*) Use this formula to show (without the calculus) that the deviation has the least value when the ray is symmetrically situated with respect to the two refracting faces. (*c*) Show that, for minimum deviation, the formula reduces to the classical laboratory expression for determining the index of refraction.

$$\frac{\sin^2\frac{1}{2}(a+d)}{\sin^2\frac{1}{2}a} = n^2 + \frac{\sin\frac{1}{2}d\,\sin(a+\frac{1}{2}d)\,\sin^2\frac{1}{2}(a+d)\,\sin^2(r-\frac{1}{2}a)}{[\sin^2\frac{1}{2}a\,\cos^2\frac{1}{2}(a+d)\,\cos^2(r-\frac{1}{2}a)+\cos^2\frac{1}{2}a\,\sin^2\frac{1}{2}(a+d)\,\sin^2(r-\frac{1}{2}a)]\,\sin^2\frac{1}{2}a}.$$

SOLUTION BY THE PROPOSER.[1]

Let i and i' denote, respectively, the angles which the segments of the ray outside the prism make with the normals at the points of incidence and emergence. Let r and r' be the corresponding angles inside the prism.

(*a*) From the geometry of the obvious diagram

$$r + r' = a$$

and

$$i + i' = a + d.$$

Put $r = \frac{1}{2}a + x$, and therefore $r' = \frac{1}{2}a - x$. Then

$$\sin i = n\sin(\tfrac{1}{2}a + x)$$

and

$$\sin i' = n\sin(\tfrac{1}{2}a - x).$$

Adding and subtracting, we have

$$\begin{aligned} 2\sin\tfrac{1}{2}(a+d)\cos\tfrac{1}{2}(i-i') &= 2n\sin\tfrac{1}{2}a\cos x, \\ 2\cos\tfrac{1}{2}(a+d)\sin\tfrac{1}{2}(i-i') &= 2n\cos\tfrac{1}{2}a\sin x; \end{aligned}$$

whence

$$n^2 = \frac{\sin^2\frac{1}{2}(a+d)\cos^2\frac{1}{2}(a+d)}{\sin^2\frac{1}{2}a\cos^2\frac{1}{2}(a+d)\cos^2 x + \cos^2\frac{1}{2}a\sin^2\frac{1}{2}(a+d)\sin^2 x}$$

or

$$\begin{aligned} \frac{\sin^2\frac{1}{2}(a+d)}{\sin^2\frac{1}{2}a} &= n^2 + \frac{[\cos^2\frac{1}{2}a\sin^2\frac{1}{2}(a+d) - \sin^2\frac{1}{2}a\cos^2\frac{1}{2}(a+d)]\sin^2\frac{1}{2}(a+d)\sin^2 x}{[\sin^2\frac{1}{2}a\cos^2\frac{1}{2}(a+d)\cos^2 x + \cos^2\frac{1}{2}a\sin^2\frac{1}{2}(a+d)\sin^2 x]\sin^2\frac{1}{2}a} \\ &= n^2 + \frac{\sin\frac{1}{2}d\sin(a+\frac{1}{2}d)\sin^2\frac{1}{2}(a+d)\sin^2 x}{[\sin^2\frac{1}{2}a\cos^2\frac{1}{2}(a+d)\cos^2 x + \cos^2\frac{1}{2}a\sin^2\frac{1}{2}(a+d)\sin^2 x]\sin^2\frac{1}{2}a}, \end{aligned}$$

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[1] See *1921*, 1. The proof and the formulæ given in this solution apply directly to the more general case of the deviation of the projected ray; that is, to D' of the paper referred to.

which is the required formula.[1]

(*b*) Since a and d are positive acute angles, the last term cannot become negative and is zero only when $x = 0$. Therefore d will have a minimum value when $x = 0$, so that $r = r' = \frac{1}{2}a$, and $i = i'$, and the complete ray is symmetrically situated with respect to the prism.

(*c*) For this minimum value of d, say d_0, we have the classical laboratory formula

$$n = \frac{\sin \frac{1}{2}(a + d_0)}{\sin \frac{1}{2}a}$$

2879 [1921, 89]. Proposed by E. J. OGLESBY, Washington Square College.

Given the values of $U_{5:9}$, $U_{5:10}$, $U_{5:11}$, $U_{6:9}$, $U_{6:10}$, $U_{6:11}$, $U_{7:9}$, $U_{7:10}$, $U_{7:11}$ where $U_{h:k} = \sqrt{hk}$, find the value of $U_{6\cdot 2:9\cdot 3}$ by interpolation.

SOLUTION BY J. B. REYNOLDS, Lehigh University.

From the first three given terms we find by the method of differences[2] $U_{5:9.3} = 6.8190$, from the next three $U_{6:9,3} = 7.4698$ and from the last three $U_{7:9.3} = 8.0685$. Using these three values then as a series by the same method we find $U_{6.2:9.3} = 7.5937$.

Also solved by H. N. CARLETON.

2881 [1921, 89]. Proposed by E. B. ESCOTT, Oak Park, Ill.

If, in the polynomial $X^3 - 2$, we substitute $x^2 + x - 4$ for X, the given expression can be factored, that is, $X^3 - 2 \equiv (x^3 + 3x^2 - 3x - 11)(x^3 - 6x + 6)$. Find a substitution for X so that the polynomial $X^3 + pX^2 + qX + r$ may be factored.

SOLUTION BY THE PROPOSER.

Let

$$X^3 + pX^2 + qX + r = (X - b)(X + c)^2 - a^2(X - d)^2. \tag{1}$$

Expanding the second member and equating coefficients of like powers of X, we get

$$a^2 + b - 2c + p = 0, \qquad 2a^2d - 2bc + c^2 - q = 0, \qquad \text{and} \qquad a^2d^2 + bc^2 + r = 0.$$

Eliminating b and d and solving for a^2 we get

$$a^2 = \frac{(3c^2 - 2pc + q)^2}{4(c^3 - pc^2 + qc - r)}\,.$$

Therefore, it is necessary and sufficient that[3]

$$c^3 - pc^2 + qc - r = n^2,$$

and we get by substitution

$$a = \frac{3c^2 - 2pc + q}{2n}, \qquad d = -\frac{c^3 - qc + 2r}{3c^2 - 2pc + q}, \qquad \text{and} \qquad b = -a^2 + 2c - p.$$

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[1] We might say

$$\begin{aligned} \frac{1}{n^2} &= \frac{\sin^2 \frac{1}{2}a \cos^2 x}{\sin^2 \frac{1}{2}(a + d)} + \frac{\cos^2 \frac{1}{2}a \sin^2 x}{\cos^2 \frac{1}{2}(a + d)} \\ &= \frac{\sin^2 \frac{1}{2}a}{\sin^2 \frac{1}{2}(a + d)} + \left[\frac{\cos^2 \frac{1}{2}a}{\cos^3 \frac{1}{2}(a + d)} - \frac{\sin^2 \frac{1}{2}a}{\sin^2 \frac{1}{2}(a + d)}\right]\sin^2 x \\ &= \frac{\sin^2 \frac{1}{2}a}{\sin^2 \frac{1}{2}(a + d)} + \frac{4 \sin \frac{1}{2}d \sin (a + \frac{1}{2}d) \sin^2 x}{\sin^2 (a + d)} \end{aligned}$$

and use this formula for (*b*) and (*c*) instead of the formula in the text, d being a minimum when $[\sin^2 \frac{1}{2}a]/[\sin^2 \frac{1}{2}(a + d)]$ is a maximum.—EDITORS.

[2] See *1921*, 330.

[3] Thus it seems to be a necessary part of the hypothesis that there is a rational number $-c$ that will make the given polynomial equal to minus the square of a rational number.

Letting f denote the polynomial, if $f(-c) = -n^2$ we can write

$$f(X) = (X + c)^2\left[X - 2c + p + \left(\frac{f'(-c)}{2n}\right)^2\right] - \left[\frac{f'(-c)}{2n}(X + c) - n\right]^2,$$

which we can make the difference of two squares by putting $X - 2c + p + \left(\frac{f'(-c)}{2n}\right)^2 = (x + t)^2$.

—EDITORS.